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PROBLEMS OF TWO-DIMENSIONAL ANALYSIS OF PARTICLE DISTRIBUTION IN AN ATMOSPHERE AEROSOL

By

K. B. Yudin

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

PROBLEMS OF TWO-DIMENSIONAL ANALYSIS OF PARTICLE DISTRIBUTION IN AN ATMOSPHERIC AEROSOL

K. B. Yudin

For studying the microphysical characteristics of an aerosol extensive use is being made of photoelectric sensors, the operation of which is based on the measurement of the light which is scattered by each particle at the moment when it passes the illuminated measuring light cavity. The pulses of scattered light, which are recorded by a photomultiplier, are converted into electrical pulses and are fed to specialized computing and recording devices. The dimensions of the light measuring cavity of the sensor are made quite small so that the probability of the simultaneous entry of two particles into it with the assigned aerosol concentration would be low.

The processing of the results of the measurements is done with the assistance of the device of mathematical statistics. In article [2], monograph [5], and a number of other sources a method has been developed for calculating the physical characteristics of an investigated aerosol medium (visibility, water content) and the coefficients of the approximating functions for the distribution of particles by size, i.e., the parameters which result from a unidimensional distribution. The contemporary level of technology makes it possible to conduct a second form of analysis -

to study the distribution of time intervals between the particles which are arriving for measurement, which carry information on the distribution of particles in space. Two-dimensional analysis makes it possible to appraise the degree of connection of particle size with the distances between them and their mutual influence, and in particular to evaluate the fluctuations of mean, mean-square and mean-cubic diameters under the influence of spatial distribution. The main part of the present work is devoted to the mathematical aspects of the analysis of two-dimensional distribution.

The basic relationships are considered which describe the distribution of a system of two random processes relative to the problem of studying the microstructure of an aerosol. On the basis of the relationships for calculation of the numerical characteristics of distribution a method is proposed for evaluating their fluctuations under the action of the spatial distribution of particles. Since the analysis of microstructure is carried out with the help of discrete specialized computers, the analogous expressions are given also in a discrete form. A separate analysis is made of a case in which the distribution of particles in space is assumed known and subordinate to Poisson statistics. In the concluding part of the article the question of the selection of the intervals of quantization and the number of channels of the analyzer in the study of particle distribution by size is considered.

1. General Relationships

The signal on the output of the photoelectric converter represents the randomly distributed in time sequence of electrical pulses of random amplitude and can be described by the two-dimensional integral law of distribution

$$F(D,\tau)=P[(D<D_i)(t<\tau)],$$

where P - probability operator, D - particle size, D_i - certain current value of particle size, t - random time interval between two particles arriving in succession for analysis, τ - certain

current value of interval t. For definiteness we will consider each of the intervals as belonging to the particle following behind it.

For practical calculations it is more convenient to characterize the distribution of a system of random variables not by the function, but by the two-dimensional density of distribution

$$f(D, \tau) = \frac{\partial^2 F(D, \tau)}{\partial D \, \partial \tau}. \qquad (1)$$

Then the function of distribution can be written in the form

$$F(D, \tau) = \int_{D_{min}}^{D} \int_{\tau_{min}}^{\tau} f(D, \tau) \, dD \, d\tau,$$

from which the densities of distribution of each of the processes are readily determined:

$$f_1(D) = \int_{\tau_{min}}^{\tau_{max}} f(D, \tau) \, d\tau \text{ и } f_2(\tau) = \int_{D_{min}}^{D_{max}} f(D, \tau) \, dD.$$

We will denote the conditional densities of distribution of variables, calculated under the condition that the second variable accepted an assigned value, through

$$\varphi_1(D/\tau) \text{ и } \varphi_2(\tau/D).$$

In order to avoid errors it is necessary to keep in mind that with a fixed, for example, particle size $D=D_1$ the conditional density $\phi_2(\tau/D_1)$ characterizes not the intevals of time between two particles of the same size D_1 which are arriving in sequence for analysis, but the intervals of time, preceding the particles with the size D_1, and following behind the preceding particles regardless of their size. In the same manner in the case of a fixed $\tau=\tau_j$ the conditional density $\phi_1(D/\tau_j)$ characterizes the distribution by size of the particles which are arriving for analysis after the interval τ_j after the preceding particle, and not the distribution of particle size on assigned intervals of time.

The density of distribution of the system is determined by the expression

$$f(D, \tau) = f_1(D)\varphi_2(\tau/D) = f_2(\tau)\varphi_1(D/\tau).$$

If the investigated variables are independent of each other (such a situation, for example, will take place in the case of strict fulfillment of the Poisson law for the distribution of particles in space), then the problem is simplified and

$$f(D, \tau) = f_1(D)f_2(\tau). \tag{2}$$

2. Numerical Characteristics of Distribution

In the course of the test the investigators, as a rule, take several measurements of the function or density of distribution of particles. Each of such measurements is called a realization, and the volume of aerosol containing a certain number of particles which is taken for analysis in one realization - a sample. In the subsequent account it is assumed that the investigated process is stationary in the course of the test.

The two-dimensional analysis of particle distribution by size and interval is made with the help of specialized computing devices - amplitude-time analyzers. Based on the distribution obtained in one realization the average, mean square, and mean cubic dimensions of the particles are calculated using the formulas

$$\overline{D}_1 = \int_{D_{min}}^{D_{max}} Df_1(D)\,dD, \quad \overline{D}_2 = \sqrt{\int_{D_{min}}^{D_{max}} D^2 f_1(D)\,dD},$$

$$\overline{D}_3 = \sqrt[3]{\int_{D_{min}}^{D_{max}} D^3 f_1(D)\,dD}. \tag{3}$$

For increasing accuracy by means of excluding errors of a statistical nature the average dimensions are frequently calculated on K realizations on the basis of the relationship

$$\tilde{D}_1 = \frac{1}{K}\sum_k \int_{D_{min}}^{D_{max}} D f_1(D)\, dD.$$

For the mean square and mean cubic dimensions correspondingly

$$\tilde{D}_2 = \sqrt{\frac{1}{K}\sum_k \int_{D_{min}}^{D_{max}} D^2 f_1(D)\, dD},$$

$$\tilde{D}_3 = \sqrt[3]{\frac{1}{K}\sum_k \int_{D_{min}}^{D_{max}} D^3 f_1(D)\, dD}.$$

The average interval between particles arriving for analysis is calculated by the formulas

$$\bar{\tau} = \int_{\tau_{min}}^{\tau_{max}} \tau f_2(\tau)\, d\tau, \qquad \tilde{\tau} = \frac{1}{K}\sum_k \int_{\tau_{min}}^{\tau_{max}} \tau f_2(\tau)\, d\tau.$$

The degree of connection can be characterized by the correlation moment, representing the second mixed central moment of a system of two random processes and determined by the expression

$$R_{D,\tau} = \int_{D_{min}}^{D_{max}} \int_{\tau_{min}}^{\tau_{max}} (D - \bar{D})(\tau - \bar{\tau}) f(D, \tau)\, dD\, d\tau.$$

As is known, the difference of the correlation moment from zero proves the dependence of the processes. Since the correlation moment, as this is evident from the relationship given, characterizes not only the degree of connection of the variables, but also their scattering, then for the evaluation of the degree of connection in a "pure form" it is convenient to switch to a coefficient of correlation, equal to

$$\eta_{D,\tau} = \frac{R_{D,\tau}}{\sigma_D \sigma_\tau},$$

where σ_D and σ_τ - mean square deviations of variables D and τ, equal to

$$\sigma_D = \sqrt{\int_{D_{min}}^{D_{max}} (D-\bar{D})^2 f_1(D)\, dD},$$

$$\sigma_\tau = \sqrt{\int_{\tau_{min}}^{\tau_{max}} (\tau-\bar{\tau})^2 f_2(\tau)\, dt}.$$

Indirectly, apparently the connection under consideration can be judged based on the coefficient of correlation between the average diameters and the concentration of particles in K realizations.

It is interesting to appraise the influence of spatial distribution of particles on the average dimensions calculated according to formulas (3). For this purpose it is possible to calculate the dispersion of average dimensions which is caused by the random distribution of time intervals between the particles which have arrived for analysis

$$\sigma_D^2(\tau) = Д\left[\int_{D_{min}}^{D_{max}} Df(D,\tau)\, dD\right] = \int_{D_{min}}^{D_{max}} D^2 Д\,|f(D,\tau)|\, dD.$$

Here and subsequently Д - dispersion operator.

With a calculation that dispersion is calculated in a point, equal to the average of one of the variables which enters into the system, it is possible to write [1]

$$\sigma_{D,}^2(\tau) = \int_{D_{min}}^{D_{max}} D^2 Д\,|\tau(D)|\, dD. \qquad (4)$$

For the mean square and the mean cube dimensions the corresponding expressions will be

$$\sigma^2_{D_2}(\tau) = \sqrt{\int_{D_{min}}^{D_{max}} D^4 Л[\tau(D)]\, Dd} \,,$$

$$\sigma^2_{D_3}(\tau) = \sqrt[3]{\int_{D_{min}}^{D_{max}} D^6 Л[\tau(D)]\, dD} \,.$$

It should be noted that the question of the mutual connection of dimensions of particles and the intervals between them, and moreso the degree of connection, has not been studied at the present time. In the overwhelming majority of works on the study of the microstructure of an aerosol only a unidimensional analysis of the distribution of particles by size is made. In this case for the calculation of the average diameters expressions (3) are used.

3. Discrete Form

For measuring the laws of distribution of particles with a two-dimensional analyzer the magnitudes of frequencies p_{ij}, corresponding to the probability of entry of particles into each of the channels of the analyzer, are defined as the ratio of the number of particles n_{ij} of diameter D_i, measured in the given channel, which have arrived for analysis during the interval of time τ_j after the preceding particle, to the overall number of all the particles recorded

$$p_{ij} = \frac{n_{ij}}{\sum_i \sum_j n_{ij}} = \frac{n_{ij}}{N} \,.$$

The probability densities of each of the processes can be found using the formulas

$$f_1(D_i)=\sum_j p_{ij}, \quad f_2(\tau_j)=\sum_i p_{ij}.$$

In the case of unidimensional analysis the magnitudes of frequencies are determined from the relationship

$$p_i=\frac{n_i}{\sum_i n_i}=\frac{n_i}{N}.$$

The average, mean square, and mean cubic diameters of one realization are calculated using the formulas

$$\bar{D}_1=\sum_i D_i p_i, \quad \bar{D}_2=\sqrt{\sum_i D_i^2 p_i}, \quad \bar{D}_3=\sqrt[3]{\sum_i D_i^3 p_i}.$$

The mean square deviation of any of the diameters

$$\sigma_{Ds}=\sqrt{\sum_i (D_{is}-\bar{D}_s)^2 p_i}.$$

where the symbol s shows the deviation of which of the diameters is calculated. For processing the results of the tests on K realizations it is convenient to introduce the following relationships:

$$\tilde{n}_i=\frac{1}{K}\sum_{l=1}^{k} n_{il}, \quad \tilde{N}=\frac{1}{K}\sum_{l=1}^{k} N_l, \quad \tilde{p}_i=\frac{1}{K}\sum_{l=1}^{k} p_{il}=\frac{\tilde{n}_i}{\tilde{N}}.$$

then

$$\tilde{D}_1=\sum_i D_i\tilde{p}_i, \quad \tilde{D}_2=\sqrt{\sum_i D_i^2\tilde{p}_i}, \quad \tilde{D}_3=\sqrt[3]{\sum_i D_i^3\tilde{p}_i},$$

$$\sigma_{Ds}=\sqrt{\frac{1}{K}\sum_{l=1}^{k}(\bar{D}_{ls}-\tilde{D}_s)^2}.$$

For the intervals of time between particles

$$\bar{\tau} = \sum_j \tau_j p_j, \qquad \sigma_\tau = \sqrt{(\tau_j - \bar{\tau})^2 p_j}\,.$$

The correlation moment is calculated using the relationship

$$R_{D\tau} = \sum_i \sum_j (D_i - \bar{D})(\tau_j - \bar{\tau})\, p_{ij}\,.$$

The correlation moment between average diameter of particles and their concentration in K realizations can be found from the relationship

$$R_{\bar{D}N} = \frac{1}{K^2} \sum_{l=1}^{k} \sum_{l=1}^{k} (D_l - \bar{D})(N_l - \bar{N}).$$

Here the doubled sum on l indicates that all the products in the case of all values of l for each of the factors should be summed. The mean square deviations are found using the formulas

$$\sigma_N = \sqrt{\frac{1}{K} \sum_{l=1}^{k} (N_l - \bar{N})^2}.$$

The dispersion of average diameter under the influence of spatial distribution of particles is equal to

$$\sigma^2_{\bar{D}_1}(\tau) = \text{Д}\left[\sum_i D_i p_i(D, \tau)\right] = \sum_i D_i^2 \text{Д}\,[p_i(D, \tau)] = \sum_i D_i^2 \text{Д}\,[p_i(\tau)].$$

For the dispersions of the mean square and mean cubic diamters the following expressions are valid

$$\sigma^2_{\bar{D}_2} = \sqrt{\sum_i D_i^4 \text{Д}\,[p_i(\tau)]}\,, \qquad \sigma^2_{\bar{D}_3} = \sqrt[3]{\sum_i D_i^6 \text{Д}\,[p_i(\tau)]}\,.$$

In the expression for average diameters, calculated on K realizations, instead of $Д[p_i(\tau)]$ we introduce $Д[\tilde{p}_i(\tau)]$.

The coefficients of variations (variability) of average diameters in all cases are calculated as the ratio of the mean square deviation to the corresponding average

$$\hat{\nu}_s = \frac{\sigma_s}{\bar{D}_s}.$$

4. Distribution According to Poisson Law

The evaluation of σ on the basis of expression (4) can be done with unidimensional analysis based on the a priori known form of distribution of particles in space. According to concepts which exist at the present time this distribution is subordinate to Poisson statistics. In this case the probability of finding n_i particles of diameter D_i in a unit measured volume is given by the expression

$$p_i = \frac{\tilde{n}_i^{n_i}}{n_i!} e^{-\tilde{n}_i}.$$

This distribution has mathematical expectation and dispersion, equal to $\tilde{n}_i$. Since the concentration of particles of different diameters is different, i.e. $\tilde{n}_i = f(D)$, then it is evident that the dispersion of intervals between particles of different diameters is different and is proportional to their concentration, i.e.

$$Д[\tau(D)] = \tilde{n}(D).$$

The last expression shows that the dispersion of average diameters can be calculated only on the basis of several realizations, and their number should be sufficient enough so that the evaluation

of ñ(D) would be significant.

Having substituted the value Д[τ(D)] into the expression, for example, of dispersion of average diameter under the influence of the spatial distribution of particles, in the case of distribution by Poisson law we obtain

$$\sigma^2_{\bar{D}_i}(\tau)_{\text{Пуас}} = \frac{1}{N} \int\limits_{D_{\min}}^{D_{\max}} D^2 \tilde{n}(D)\, dD. \tag{5}$$

In the same manner the expression for mean square and mean cubic diameters is obtained.

In a discrete form, taking into account that $p_i = \frac{n_i}{N}$, we obtain

$$Д[p_i(\tau)] = \frac{Д[n_i]}{N^2} = \frac{\tilde{n}_i}{N^2} = \tilde{p}_i \frac{N}{N^2}$$

and for K realizations

$$Д[\tilde{p}_i(\tau)] = \frac{Д[\tilde{n}_i]}{\tilde{N}^2} = \frac{Д[n_i]}{K\tilde{N}^2} = \frac{\tilde{n}_i}{K\tilde{N}^2} = \frac{p_i}{K\tilde{N}}.$$

Dispersion and the coefficient of variations of average diameter in K realizations in the case of Poisson distribution of particles in space will be equal to

$$\sigma^2_{\bar{D}_i}(\tau)_{\text{Пуас}} = \frac{1}{K\tilde{N}} \sum_k D_i^2 p_i,$$

$$\delta_{\bar{D}_i}(\tau)_{\text{Пуас}} = \frac{1}{\sqrt{K\tilde{N}}} \frac{\sqrt{\sum_k D_i^2 \tilde{p}_i}}{\sum_k D_i \tilde{p}_i}. \tag{6}$$

These parameters can also be calculated using experimental data and the following relationships:

$$\sigma^2_{\bar{D}_1\, \text{опыт}} = \frac{1}{K}\sum_k (\bar{D}_{1k} - \tilde{D}_1)^2, \quad \text{где} \quad \tilde{D}_1 = \frac{1}{K}\sum_k \bar{D}_{1k}.$$

And if it turns out **that** $\sigma_{\bar{D}_1}(\tau)_{\text{Пуас}} \geqslant \sigma_{\bar{D}_1\, \text{опыт}}$, then the fluctuations of D_1 can be explained by fluctuations in the distribution of particles in space. **When** $\sigma_{\bar{D}_1\, \text{Пуас}} < \sigma_{D_1\, \text{опыт}}$ and correspondingly $\delta_{\bar{D}_1\, \text{Пуас}} < \delta_{\bar{D}_1\, \text{опыт}}$, it follows to assume that the fluctuation of **average diameter is** connected with fluctuations in the distribution of particles by size. What was said is also valid for the mean square and mean cubic diameters and the characteristics of an aerosol which are calculated on their basis.

Let us consider, for example, the fluctuation in water content of an aerosol with an unchanged distribution of particles by size. The water content of the aerosol is calculated using the formula

$$q = \frac{\pi}{6}\sum_k D_i^3 n_i = \frac{\pi}{6}\tilde{N}\sum_k D_i^3 p_i = \frac{\pi}{6}\tilde{N}\tilde{D}_3^3.$$

The dispersion of water content in K realizations, caused by the fluctuation in the number of particles due to their random distribution in space, will be determined by the expression

$$\sigma^2_{\tilde{q}} = Д\left[\frac{\pi\tilde{N}}{6}\tilde{D}_3^3\right] = \left(\frac{\pi\tilde{N}}{6}\right)^2 Д[D_3^3] = \left(\frac{\pi\tilde{N}}{6}\right)^2 \sigma^6_{\tilde{D}_3}(\tau).$$

In the case of distribution of particles in space according to Poisson law this expression acquires the form

$$\sigma^2_{q\, \text{Пуас}} = \left(\frac{\pi\tilde{N}}{6}\right)^2 \frac{\sum_k D_i^6 p_i}{K\tilde{N}},$$

i.e., dispersion is proportional to $\tilde{N}$. The corresponding coefficient of variation

$$\delta_{\bar{q}\,\text{Пуас}} = \frac{1}{\sqrt{K\tilde{N}}} \frac{\sqrt{\sum_{k} D_i^6 p_i}}{\sum_{k} D_i^3 \tilde{p}_i}.$$

5. Intervals of Quantization and Number of Channels for the Analyzer

An important methodical problem in the study of the distribution of particles by size is the rational selection of the intervals of quantization and the number of channels for the threshold analyzer. Below we will consider one of the possible methods of evaluating these parameters which is based on knowledge of the law of measured distribution and permissible error. For this purpose the density of probability $p_i = f_i(D_i)$ is approximated by a polynomial of the first degree for each interval $D_{i+1} - D_i$ (piecewise linear approximation). The error of such approximation is determined by the remainder term of the interpolation formula. For the Newton formula the maximum magnitude of the remainder term is equal to [4]

$$|J(D_i)|_{max} = \frac{f''(D_i)}{2!} \left(\frac{D_{i+1} - D_i}{2}\right)^2.$$

Assuming the absolute error $\Delta p_{i\ max} = {}_{max}|J(D_i)|$, for determination of the sector of partitioning $\Delta D = (D_{i+1} - D)$ in [3, 6] the following expression is obtained

$$\Delta D = \sqrt{\frac{p_i}{|f''(D)|_{max}}}.$$

where $|f''(D)|max$ - maximum absolute value of the second derivative in point D of the interval $D_{i+1}-D_i$ or for relative error

$$\varepsilon = \frac{\Delta p}{f(D)}, \qquad \Delta D = \sqrt{8\varepsilon \frac{f(D)}{|f''(D)|_{\max}}}.$$

The required number of channels for the analyzer with a uniform partitioning by particle size will be equal to

$$m > \frac{D_{\max} - D_{\min}}{\Delta D}.$$

For example, for gamma-distribution with α whole and positive

$$f(D) = \frac{D^{\alpha}}{\alpha! \beta^{\alpha+1}} e^{-\frac{D}{\beta}},$$

$$f''(D) = \frac{D^{\alpha-2}}{\alpha! \beta^{\alpha+1}} \left[\left(\alpha - \frac{1}{\beta} D\right)^2 - \alpha\right] e^{-\frac{D}{\beta}},$$

$$\frac{f(D)}{|f''(D)|} = \frac{D^2}{\left|\left(\alpha - \frac{D}{\beta}\right)^2 - \alpha\right|}.$$

With the parameters of gamma-distribution $\alpha=8$ and $\beta=1.2$ which are encountered frequently in clouds the second derivative is maximum in a point which is close to 4.5 μm. If the range of measured dimensions of particles lies within the limits from 2 to 30 μm, then with a relative error $\varepsilon=0.05$ we have

$$\frac{f(4{,}5)}{|f''(4{,}5)|} = 2{,}3, \quad \Delta D = 0{,}96, \quad m = 31.$$

In this case the error of quantization in all the remaining intervals will be less than assigned. If with the assigned accuracy it is required to determine the distribution in the area of average size $D=\beta(\alpha+1)=10.8$ μm, then

$$\frac{f(10,8)}{|f''(10,8)|}=16,7, \quad \Delta D=2,6 \overset{\mu m}{\text{мкм}}, \quad m=12,$$

but the error in the area of sizes of 4.5 μm increases to 0.36.

With nonuniform quantization the magnitudes of the intervals ΔD are not equal to each other, but change according to a selected law. It is convenient to consider the advantages of nonuniform quantization in an example of quantization according to logarithmic law. It is known that gamma-distribution for the description of distribution by size of cloud particles is introduced as the approximating function for logarithmically normal distribution. Having designated $u=\ln D$, we have

$$\psi_{(u)}=\frac{1}{\sqrt{2\pi}\,\sigma_u}e^{-\frac{(u-u_0)^2}{2\sigma_u^2}}.$$

The parameters of logarithmically normal distribution, corresponding to the gamma-distribution which was considered earlier, are approximately equal to $u_0=2.38$, $\sigma_u=0.9$, $0.69\leqslant u\leqslant 3.47$. Using the method which was set forth earlier, we will divide the distribution uniformly on logarithms D

$$\psi''(u)=\frac{1}{\sqrt{2\pi}\,\sigma_u^3}e^{-\frac{(u-u_0)^2}{2\sigma_u^2}}\left[\left(\frac{u-u_0}{\sigma_u}\right)^2-1\right],$$

$$\frac{\psi(u)}{\psi''(u)}=\frac{\sigma_u^2}{\left(\frac{u-u_0}{\sigma_u}\right)^2-1}.$$

$\psi''(u)$ is maximum in points $u_{1,2}=u_0\pm\sigma_u\sqrt{3}$, then

$$\frac{\psi(u)}{|\psi''(u)|_{max}} = \frac{\sigma_u^2}{2} = 0{,}45, \quad \Delta u = 0{,}42, \quad m_u = 7.$$

The thresholds of the analyzer channels, expressed in particle dimensions (μm), and not their logarithms, should be set on the levels 2.0, 2.9, 4.5, 6.6, 9.9, 14.8, 22.0, 32.9.

Thus due to the expansion of channels in the area of large particles the required number of channels was reduced to 7, and the error from quantization was not increased.

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